Compact quantum groups and their representation categories

Let G be a compact group. The C*-algebra $A = C(G)$ of continuous functions on G has an additional structure defined by the multiplication in G, the homomorphism

$$\Delta\colon A \to A \otimes A = C(G \times G), \quad \Delta(f)(g,h) = f(gh).$$

The associativity of the multiplication implies that

$$(\Delta \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \Delta) \circ \Delta \tag{1}$$

as homomorphisms $A \to A \otimes A \otimes A$. On the other hand, the existence of the inverse can be shown to imply that

$$\text{the spaces } \ (A \otimes 1)\Delta(A) \ \text{ and } \ (1 \otimes A)\Delta(A) \ \text{ are dense in } \ A \otimes A. \tag{2}$$

It turns out that any pair (A, Δ), where A is a unital commutative C*-algebra and $\Delta\colon A \to A \otimes A$ is a homomorphism satisfying properties (1) and (2), arises from a compact group as described above. If however we do not assume commutativity of A, we get a completely new class of objects, called compact quantum groups.

The goal of the coarse is to develop the theory of compact quantum groups and their duals, so called discrete quantum groups, introduce some examples, in particular the q-deformations of Lie groups (with a detailed study of the deformation of the simplest compact Lie group $SU(2)$), and then study quantum group actions, boundary theory and differential geometry, emphasizing the role of categories of representations in the process.

The prerequisite is a basic course in operator algebras. Some acquaintance with Lie groups, representation theory and category theory will be helpful but is not assumed.